A Comparative Study on π and T Equivalent Models for the Analysis of Transformer Ferroresonance

Saeed Jazebi, *Student Member, IEEE*, Ashkan Farazmand, Brahadeesh Perinkolam Murali, and Francisco de León, *Senior Member, IEEE*

***Abstract*—The performance of the T and the π equivalent models used to represent transformers are tested under ferroresonance. Comparisons between simulations and laboratory experiments show the superiority of the π equivalent circuit.**

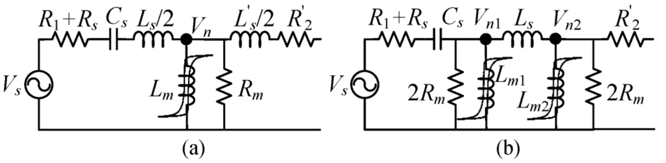
***Index Terms*—Ferroresonance, transformer modeling.**

I. Introduction

Ferroresonance may cause severe temporary overvoltages and damage the internal or external insulation of transformers. To predict possible overvoltages, proper modeling of ferroresonance is required for computer simulation. The T equivalent circuit is the most common representation of a two-winding transformer [1]. [See Fig. 1(a)]. An alternative, the π model [2], is a duality derived representation for a transformer that has advantages over the T model. [See Fig. 1(b)]. In this paper, the T and π models are compared using time-domain simulations against laboratory experiments. The results show that the T model may produce large errors while the π model properly predicts the occurrence of ferroresonance. All simulations in this letter are carried out with the Electromagnetic Transients Program (EMTP) considering detailed representation of the hysteresis curves (except when noted) including nonlinear magnetization and losses.

II. Simulations Versus Laboratory Experiments

Two 1-kVA, 120:120-V transformers (T1 and T2) with electrical parameters presented in Table I are selected. T1 has typical impedance parameters for a small power transformer, while T2 has been selected because it has a substantially larger leakage inductance and serves to accentuate the differences between the two circuits. The equivalent circuits for the experimental setup are depicted in Fig. 1. The parameters are obtained from the standard impedance and open-circuit tests according to the IEEE Standard C57.12.91-1995.



(a) (b)

Fig. 1. Equivalent circuit of the experimental setup. (a) π model. (b) T model.

TABLE I
Electrical Parameters of Transformers

Code	R_1 (Ω)	R'_2 (Ω)	R_m (Ω)	L_s (mH)	$L_{m\text{-}lin}$ (mH)	$L_{m\text{-}sat}$ (mH)
T1	0.277	0.300	1,415.9	0.23	1,284.7	316
T2	0.306	0.305	1,074.7	8.78	1,669.6	463

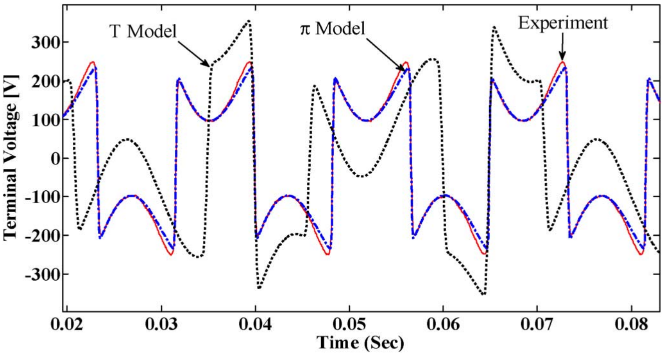


Fig. 2. Terminal voltage of T2 with 20-μF series capacitance.

A large number of experiments have been carried out with the secondary of the transformers open-circuited and applying rated voltage. Ferroresonance is chaotic and depends on initial conditions. To obtain consistent results, the core was demagnetized, and the series capacitor was discharged before each experiment. We made sure that the results were consistent, and not affected by the chaotic nature of ferroresonance. Only three cases are discussed here. The first test is on T1 when a 20-μF capacitance was connected in series with the terminals. Both models show the occurrence of ferroresonance with voltages within a few percent error when compared to the experiments. (Details are not presented.)

In the second experiment, ferroresonance occurs on T2 with the series 20-μF capacitance. (See Figs. 2 and 3.) Note, however, that the T model exhibits a completely different behavior than the measurements. The mismatch is evident in both voltage and current; and even the frequency of oscillation is different. The computed overvoltage is 44% higher than the experimental result. On the other hand, the current and voltage of the π model are visibly correct with a relative difference of maximum of about 5% with respect to the experimental results.

The third experiment presents ferroresonance between T1 and a 60-μF capacitance. The voltage waveforms are presented in Fig. 4. One can note that the experiments and the π model

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The authors are with the Department of Electrical and Computer Engineering, Polytechnic Institute of New York University, Brooklyn, NY, 11201 USA (e-mail: jazebi@ieee.org; afaraz01@students.poly.edu; brahadeesh.murali@gmail.com; fdeleon@poly.edu).

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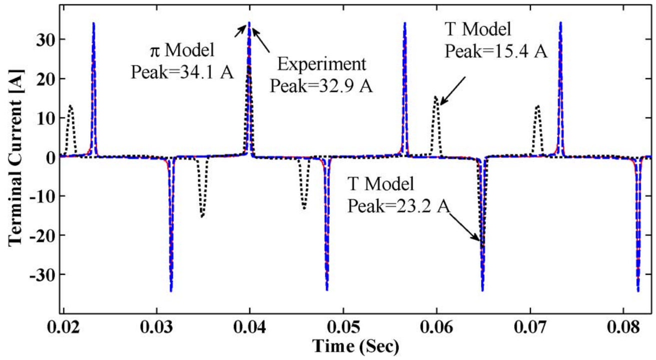

Fig. 3. Terminal current of T2 with 20-μF series capacitance.

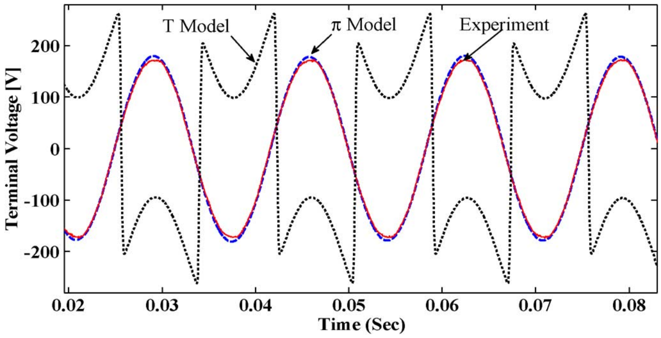

Fig. 4. Terminal voltage of T1 with 60-μF series capacitance.

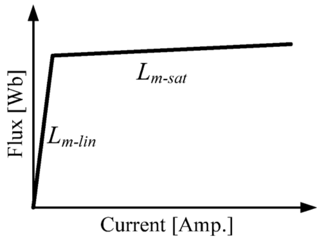

Fig. 5. Simplified magnetizing curve for T2 used for analysis purposes.

show a normal operating condition (no ferroresonance), but the T model predicts ferroresonance.

III. DISCUSSION

During ferroresonance, transformers have transitions between the linear and the nonlinear regions of the hysteresis curve. In this section, to study the performance of the transformers, the nonlinearities are represented by piecewise-linear models with only two sections. (See Fig. 5.) Parameters $L_{m-\mathrm{lin}}$ and $L_{m-\mathrm{sat}}$ are the slopes of the linear and deep saturation parts of the magnetizing curve, respectively.

In the π model, two shunt magnetizing branches exist with internal nodal voltages denoted as V_{n1} and V_{n2}. The (internal) voltage of the T model's magnetizing branch is V_n. (See Fig. 1.) The relations between the internal node voltages and the source voltage, neglecting all damping components, are

$$\frac{V_n}{V_s} = \frac{2C_sL_m\omega^2}{C_s(L_s+2L_m)\omega^2 - 2} \tag{1}$$

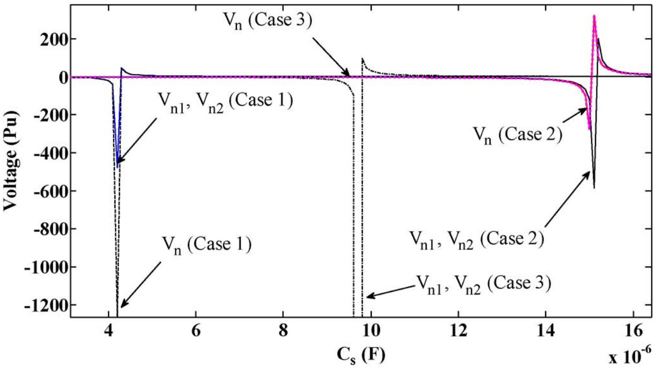

Fig. 6. Nonlinear branch voltages by varying the series capacitance for T2.

$$\frac{V_{n1}}{V_s} = \frac{C_sL_{m1}(L_s+L_{m2})\omega^2}{C_sL_{m1}(L_s+L_{m2})\omega^2 - L_{m1} - L_{m2} - L_s} \tag{2}$$

$$\frac{V_{n2}}{V_s} = \frac{C_sL_{m1}L_{m2}\omega^2}{C_sL_{m1}(L_s+L_{m2})\omega^2 - L_{m1} - L_{m2} - L_s}. \tag{3}$$

To highlight the difference in the resonance behavior of the two equivalent circuits, three cases are investigated:

Case 1) $L_m = 0.5L_{m1} = 0.5L_{m2} = L_{m-\mathrm{lin}}$ (nonsaturated conditions);
Case 2) $L_m = 0.5L_{m1} = 0.5L_{m2} = L_{m-\mathrm{sat}}$ (saturated conditions);
Case 3) $L_m = 0.5L_{m1} = L_{m-\mathrm{sat}}, 0.5L_{m2} = L_{m-\mathrm{lin}}$ (L_m and L_{m1} saturated and L_{m2} nonsaturated).

The saturation status (instantaneous flux) depends on the instantaneous voltages applied to the nonlinear inductances [See (1) to (3).] In the first case, it is assumed that both models are working in the linear part of the magnetizing curve. The second case is when T and π models are saturated. Due to the leakage inductance between the magnetizing branches in the π model, there are differences between V_{n1} and V_{n2}. Differences become more noticeable for transformers with large leakage inductance. Thus, it is possible that L_{m1} goes into saturation while L_{m2} is still working in its linear part; this situation corresponds to Case 3. For transformer T2, the terminal voltage versus the value of the series capacitance is presented in Fig. 6. The figure shows that the resonance behavior of T and π models is quite different at various operating conditions. This can also be observed from the capacitance values that would produce resonance

$$C_\pi = \frac{L_{m1}+L_{m2}+L_s}{L_{m1}(L_s+L_{m2})\omega^2} \qquad C_T = \frac{2}{(L_s+2L_m)\omega^2} \tag{4}$$

where L_m, L_{m1}, and L_{m2} can be substituted by $L_{m-\mathrm{lin}}$ or $L_{m-\mathrm{sat}}$ depending on the values of the instantaneous voltages V_n, V_{n1}, and V_{n2}. Note that the differences between C_π and C_T become larger for transformers with higher leakage inductance. For transformer T1, the resonance response of the models is much closer than for T2 (results not shown). However, sometimes the T model fails; Fig. 4 shows a case when the T model predicts ferroresonance when it does not occur in reality.

A comprehensive sensitivity analysis on transformer parameters (L_s/L_m) with respect to terminal behavior of both models for the calculation of inrush currents is presented in [3].

IV. CONCLUSION

This letter has shown that the T model may fail to reproduce ferroresonance measurements, while the π model predicts the measurements adequately in all tested cases.

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